On the Connection of Nonrenormalizable and Renormalizable Unified Nonlinear Spinor Field Models

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The nonrenormalizable first order derivative nonlinear spinor field equation with scalar interaction possesses two equivalent Hamiltonians. The first is the conventional one while the second is a two-field Hamiltonian with the original field and its parity transform. By quantization the latter leads to an inequivalent representation compared with the former. This is connected with parity symmetry breaking and the loss of simultaneous diagonalization of energy and subfield particle numbers. The corresponding grand canonical Hamiltonian is shown to result equivalently from a renormalizable second order derivative nonlinear spinor field equation. This is achieved by means of a theorem about the decomposition of higher order derivative nonlinear spinor field equations derived previously.

Unified nonlinear spinor field (≡ NSF) models are quantum field theories in which all observable (elementary and non-elementary) particles are assumed to be bound states of elementary fermion fields. Accordingly such models must be formulated by dynamical laws for selfcoupled fermion fields only.

If the dynamical law for such selfcoupled fermion fields is given by a NSF equation with first order derivatives (≡ FDNSF) and local interactions and if canonical quantization is applied, the corresponding quantum field theory is nonrenormalizable. As nonrenormalizability prevents the derivation of finite numerical results, it seems that the unified FDNSF models are meaningless. On the other hand, first order derivative models represent a distinguished class of models due to the mathematical simplicity of their field equations. Furthermore such a field law allows a physical interpretation as a covariant derivative in a Riemann-Cartan space T_4 within the framework of a Poincaré gauge field theory of gravitation for fermions, cf. Hehl, von der Heyde, Kerlick and Nester [1]. Hence this field law is obviously fundamental and in the development of quantum field theory numerous attempts have been made to give these models in spite of their nonrenormalizability a physical and mathematical meaning. Among these attempts are methods which introduce cut-offs, nonlocal interactions, lattice approximations and other arbitrary regularization procedures. Such methods have in common that they impose ad hoc manipulations on a quantum field theory which are not contained in the theoretical basis of this theory. Therefore their theoretical value is dubious and an approach is needed which does not make use of such manipulations.

The only attempt which seems to be meaningful within the systematic framework of a quantum field theory is the investigation of the problem whether under certain relaxed conditions such as symmetry breaking etc., a map can be found which allows the transition from a nonrenormalizable field theory on to a renormalizable one.

In principle this approach has to be based on the idea that nonrenormalizability is not absolute but only relative to certain representations while other appropriate and with the former inequivalent representations are renormalizable. The latter point of view was not sufficiently observed in the literature, rather the claim for giving a proof of equivalence dominated.

In the development of such "equivalence" proofs the physical background has been the fusion of bosons from fermions. If bosons are composed of fermions then the renormalizable boson-fermion coupling theories (≡ FBCT) must result from unified fermion models and if the latter are nonrenormalizable then there must be a mapping between a nonrenormalizable unified fermion model and the renormalizable fermion-boson models. It has to be emphasized that this idea is absolutely correct. The dubious part is only the peculiarity of

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claiming for equivalence of both models which is not necessarily implied by performing a map, i.e. the difference between biunique and unique maps is essential.

The idea of fermion fusion was inaugurated by de Broglie [2] who assumed the photon to be fusioned from two neutrinos. Later on, Fermi and Yang [3] proposed the pion to be a composite particle formed by a nucleon-antinucleon bound state. Finally, in the NSF approach of Heisenberg [4] any existing boson is assumed to be a bound state of elementary fermions. These authors, however, did not try to systematically establish a connection between the selfcoupled fermion fields and the corresponding FBCT. The first step in this direction was taken by Jouvet [5] who studied equivalence proofs by considering the four fermion interaction as the limit of a boson exchange interaction between fermionic currents. In the further evaluation of this approach bilinear spinorfield expressions were identified with boson operators, and several authors, Bjorken [6], Bialynicki-Birula [7], Lurie and Macfarlane [8], Guralnik [9], established a map of the Green function equations of selfcoupled fermion fields and those of quantum electrodynamics. Further progress was made by the application of the path integral representation of quantum field theory. Using the technique of auxiliary fields which was introduced by Coleman, Jackiw and Politzer [10] and Gross and Neveu [11], Kugo [12] and Kikkawa [13] derived by means of functional integration equivalent path integral expressions for selfcoupled fermion fields and FBCT. This method was improved by Eguchi [14] and extended by several authors, Saito and Shigemoto [15], Konisi, Miyata, Saito and Shigemoto [16], to establish equivalences between fermion selfcoupled theories and electroweak fermion-boson models. Similar approaches were made by Tamvakis and Guralnik [17], Campbell, Cooper, Guralnik and Snyderman [18], Haymaker and Cooper [19], Cooper, Guralnik, Haymaker and Tamvakis [20], Rembiesa [21]. Thus it seems as if the biunique map between nonrenormalizable FDNSF and renormalizable FBCT is well understood at present. Unfortunately, however, this is not the case as there remain open problems which are concerned with the conditions under which such equivalences are proved. In all these approaches it is necessary to introduce cutoffs, etc., to avoid divergencies. In this way one arrives finally at the level of ad hoc regularizations and one cannot claim to have shown a true equivalence. This point of view was systematically discussed by Kerler [22] and reemphasized by him, Kerler [23]. He investigated in detail the drawbacks of these methods and summarized that a consistent biunique map of the above kind is impossible. This was confirmed by results of Eguchi [24] who discussed subsidiary conditions with respect to such maps.

Apart from these technical problems there is an additional basic objection: If anyone really succeeded in constructing a mathematically rigorous biunique mapping from a NSF theory on to a renormalizable FBCT, then a NSF theory would degenerate to a stenography of a FBCT, i.e. it would be deprived of any remarkable physical content. Thus if one likes to consider the spinorfield as a fundamental entity one is necessarily forced to consider a biunique mapping on to FBCT as a misleading approach which obscures the true physical content of the fundamental fermion field equation. In particular if the bosons are bound states (as in NSF) then under suitable kinematical conditions their behaviour will differ from that of elementary bosons (in FBCT). Hence one is induced to look for other possibilities of obtaining inequivalent representations with respect to the FDNSF equation which allow a renormalization of this field.

One way is the introduction of indefinite metric. Indefinite metric was advocated by Heisenberg [4] in his attempt to develop a FDNSF theory of elementary particles. Indefinite metric is able to remove singularities of quantum fields which are nonrenormalizable in positive definite metric state spaces. However, indefinite metric necessitates a special treatment in order to avoid difficulties with the statistical interpretation of the theory. Concerning this problem in Heisenberg's approach no satisfactory answer was given so far. In order to achieve a renormalizability of FDNSF models Heisenberg introduced a dipole ghost regularization ad hoc. But neither the occurrence of dipole ghosts could selfconsistently be justified nor was it possible to follow the ramifications of dipole ghosts in bound states and their influence on the metric, etc. In addition, the dipole ghost assumption complicated all dynamical calculations and considerations so that there is no hope to make any progress in this direction. A way out of this difficulty is the use of higher order derivative nonlinear spinor field (≡ HDNSF) equations. Such equations exhibit selfregularizing properties and are thus renormalizable. In contrast to the ad hoc dipole ghost regularization of FDNSF equations the HDNSF equations can be treated in the framework of conventional canonical quantization apart, of course, of the problem of indefinite metric.

Higher order derivative field equations were first introduced by Bopp [25] in constructing a selfregularizing classical electrodynamics. Later on Podolski [26] made the same proposal. Wildermuth [27] first discussed the canonical quantization of higher order linear spinor field equations. A survey of the further development would, however, exceed the scope of this paper. Therefore we refer to a preceding paper [28].

HDNSF equations are of physical interest if it is possible to solve the problem of indefinite metric. In this respect some progress has been made recently. First it was shown that it is possible to derive for such equations a relativistic state representation theory in the Schrödinger picture. This is based on a decomposition theorem of higher order field equations, cf. [29] and Grosser [30], and on the exact energy eigenvalue functional equation which follows by using this theorem, Grosser, Hailer, Hornung, Lauxmann, and Stumpf [31]. Secondly by means of this state representation the two fermion sector of a second order derivative NSF was treated by the author [32] and it was shown that under certain conditions the influence of the indefinite metric on the norm of two-fermion states can be neglected. This procedure was performed in such a way that an extension seems possible to the treatment of higher fermion sectors leading to similar results. So there is a reasonable hope that a statistical consistent interpretation of HDNSF is realizable.

If we anticipate that the condition for a meaningful statistical interpretation of such a theory is satisfied, the physical meaning and the physical content of such unified field models will be of utmost interest. As a very first step in this direction one could therefore speculate about the physical meaning of the dynamical law, i.e. the HDNSF equation itself. From an axiomatic point of view such higher order equations are not so appealing. Rather one would prefer to start with a more elementary dynamical law, for instance a FDNSF equation as indicated above. Thus we investigate the problem whether it is possible to construct a map from a nonrenormalizable FDNSF equation on to a selfregularizing renormalizable HDNSF equation. It will be shown in the following that it is possible to establish such a map which is connected with the introduction of inequivalent representations of the first order equation obtained by symmetry breaking. In this way a connection between the nonrenormalizable first order equation and a renormalizable second order equation can be derived. The essential point in establishing this connection is to avoid the claim for a biunique map which is prevented by the afore-mentioned symmetry breaking.

For completeness it should be noted that in a series of papers Dürr [33] and Saller [34] tried to establish a biunique map between renormalizable HDNSF and renormalizable FBCT. Similar problems were also treated by Munczek and Nemirovsky [35] and Amati and Veneziano [36]. Such maps between renormalizable theories are, however, not the topic of our investigation and need thus not to be discussed here.

Concerning the method to establish a connection between nonrenormalizable FDNSF and renormalizable HDNSF we proceed as follows:

We first consider a FDNSF with scalar interaction. With respect to the spinorfield operator $\psi(\boldsymbol{r}, t)$ we define its spinorial parity transform $\xi(\boldsymbol{r}, t) = i\gamma_4 \psi(\boldsymbol{r}, t)$. We then show that the FDNSF possesses two equivalent Hamiltonians. The first is the conventional one, while the second is a two-field Hamiltonian with the original field and its parity transform as field variables. However, in spite of the equivalence of these Hamiltonians the corresponding Lagrangians differ from each other. By quantization these Lagrangians then lead to inequivalent representations, and the constraint $\xi(\boldsymbol{r}, t) = i\gamma_4 \psi(\boldsymbol{r}, t)$ which is fulfilled for the classical fields, cannot be maintained for the quantum fields. This is connected with parity symmetry breaking and the loss of simultaneous diagonalization of energy and subfield particle numbers. The corresponding grand canonical Hamiltonian is shown to result equivalently from a renormalizable second order derivative NSF equation. This is achieved by means of a theorem about the decomposition of HDNSF equations derived previously by the author [29] and Grosser [30].

1. Equivalent Hamiltonians

For deriving the indicated connection we start with the FDNSF equation

$$i\gamma^\mu \partial_\mu \psi(x) = g\,\psi(x)\,\bar\psi(x)\,\psi(x) \tag{1.1}$$

with scalar interaction. This interaction is the simplest one. An extension of our formalism to more complicated interactions is at present not known. The Lagrangian corresponding to (1.1) reads

$$L[\psi] := \frac{i}{2}\int [\bar\psi(x)\,\gamma^\mu\,\partial_\mu\psi(x) - \partial_\mu\bar\psi(x)\,\gamma^\mu\,\psi(x)]\,\mathrm{d}^3x - g\int[\bar\psi(x)\,\psi(x)]^2\,\mathrm{d}^3x \tag{1.2}$$

and the Hamiltonian

$$H[\psi] = -\frac{i}{2}\int[\bar\psi(\boldsymbol r)\,\boldsymbol\gamma\cdot\nabla\psi(\boldsymbol r) - \nabla\bar\psi(\boldsymbol r)\cdot\boldsymbol\gamma\,\psi(\boldsymbol r)]\,\mathrm{d}^3r + g\int[\bar\psi(\boldsymbol r)\,\psi(\boldsymbol r)]^2\,\mathrm{d}^3r \tag{1.3}$$

with $\psi(\boldsymbol r) := \psi(\boldsymbol r, 0)$. If we define the parity operation

$$\psi'(\boldsymbol r', t) = i\,\gamma_4\,\psi(\boldsymbol r, t) \tag{1.4}$$

with $\boldsymbol r' := -\boldsymbol r$, then the Lagrangian (1.2) as well as the Hamiltonian (1.3) are forminvariant under this operation, i.e. we have

$$L[\psi] = L[\psi']\,;\quad H[\psi] = H[\psi']\,. \tag{1.5}$$

This forminvariance can easily be verified by observing the transformation properties of the kinetic term and the interaction term in (1.2) and (1.3) under parity transformations. By the following theorem we obtain an equivalent two-field Hamiltonian.

Theorem: Let the field $\xi(\boldsymbol r)$ be the spinorial parity transform

$$\xi(\boldsymbol r) := \psi'(\boldsymbol r') \equiv \psi'(-\boldsymbol r) = i\,\gamma_4\,\psi(\boldsymbol r) \tag{1.6}$$

of the original field $\psi(\boldsymbol r)$, then the Hamiltonian (1.3) can be equivalently expressed by

$$\begin{aligned} H[\psi] = H[\psi,\xi] &\qquad\qquad (1.7)\\ := -\frac{i}{4}&\int[\bar\psi(\boldsymbol r)\,\boldsymbol\gamma\cdot\nabla\psi(\boldsymbol r) - \nabla\bar\psi(\boldsymbol r)\cdot\boldsymbol\gamma\,\psi(\boldsymbol r)]\,\mathrm{d}^3r\\ +\frac{i}{4}&\int[\bar\xi(\boldsymbol r)\,\boldsymbol\gamma\cdot\nabla\xi(\boldsymbol r) - \nabla\bar\xi(\boldsymbol r)\cdot\boldsymbol\gamma\,\xi(\boldsymbol r)]\,\mathrm{d}^3r\\ +\frac{1}{4}&g\int\{[\bar\psi(\boldsymbol r)+\bar\xi(\boldsymbol r)][\psi(\boldsymbol r)+\xi(\boldsymbol r)]\}^2\,\mathrm{d}^3r\,. \end{aligned}$$

Proof: We first consider the kinetic term. We have

$$\begin{aligned}\int\bar\xi(\boldsymbol r)\,\boldsymbol\gamma\cdot\nabla_r\xi(\boldsymbol r)\,\mathrm{d}^3r &= \int\bar\psi'(\boldsymbol r')\,\boldsymbol\gamma\cdot\nabla_r\psi'(\boldsymbol r')\,\mathrm{d}^3r\\ &= \int\bar\psi(\boldsymbol r)\,\gamma_4(-i)\,(\boldsymbol\gamma\cdot\nabla_r)\,\gamma_4(i)\,\psi(\boldsymbol r)\,\mathrm{d}^3r\\ &= -\int\bar\psi(\boldsymbol r)\,\boldsymbol\gamma\cdot\nabla_r\psi(\boldsymbol r)\,\mathrm{d}^3r\,.\end{aligned} \tag{1.8}$$

The same result can be obtained by a direct parity transformation of the kinetic term of (1.3) where in contrast to the derivation of Eqs. (1.5) no change of the integration variable $\boldsymbol r$ is performed. Concerning the interaction term, we observe that

$$\begin{aligned}\bar\psi'(\boldsymbol r')\,\psi'(\boldsymbol r')\,\bar\psi'(\boldsymbol r')\,\psi'(\boldsymbol r') &= \bar\psi(\boldsymbol r)\,\psi(\boldsymbol r)\,\bar\psi(\boldsymbol r)\,\psi(\boldsymbol r)\\ \bar\psi'(\boldsymbol r')\,\psi'(\boldsymbol r')\,\bar\psi(\boldsymbol r)\,\psi(\boldsymbol r) &= \bar\psi(\boldsymbol r)\,\psi(\boldsymbol r)\,\bar\psi(\boldsymbol r)\,\psi(\boldsymbol r), \qquad (1.9)\\ \bar\psi(\boldsymbol r)\,\psi'(\boldsymbol r')\,\bar\psi(\boldsymbol r)\,\psi(\boldsymbol r) &= i\,\bar\psi(\boldsymbol r)\,\gamma_4\psi(\boldsymbol r)\,\bar\psi(\boldsymbol r)\,\psi(\boldsymbol r),\\ \bar\psi'(\boldsymbol r')\,\psi(\boldsymbol r)\,\bar\psi(\boldsymbol r)\,\psi(\boldsymbol r) &= -i\,\bar\psi(\boldsymbol r)\,\gamma_4\psi(\boldsymbol r)\,\bar\psi(\boldsymbol r)\,\psi(\boldsymbol r),\end{aligned}$$

i.e. the first and the second term are equal, while the third and the fourth term have opposite sign. Similar relations can be derived for the other terms of the interaction part of (1.7). If these relations are summed up due to cancellations the interaction part of (1.3) results. Q.E.D.

Obviously (1.7) is equivalent to (1.3), provided that the subsidiary condition (1.6) is satisfied. It is, however, remarkable that a similar equivalence for the corresponding Lagrangians does not exist. Rather the Lagrangian corresponding to (1.7) reads

$$\begin{aligned}L[\psi,\xi] := &\frac{i}{4}\int[\bar\psi(x)\,\gamma^\mu\,\partial_\mu\psi(x) - \partial_\mu\bar\psi(x)\,\gamma^\mu\,\psi(x)]\,\mathrm{d}^3x\\ &-\frac{i}{4}\int[\bar\xi(x)\,\gamma^\mu\,\partial_\mu\xi(x) - \partial_\mu\bar\xi(x)\,\gamma^\mu\,\xi(x)]\,\mathrm{d}^3x\\ &-\frac{1}{4}g\int\{[\bar\psi(x)+\bar\xi(x)][\psi(x)+\xi(x)]\}^2\,\mathrm{d}^3x\,.\end{aligned} \tag{1.10}$$

With $\varphi_1 := \psi$, $\varphi_2 := \xi$ and $\lambda_1 = 2$, $\lambda_2 = -2$ we can rewrite (1.10) into the convenient form

$$\begin{aligned}L[\varphi_1,\varphi_2] &\qquad\qquad (1.11)\\ := \frac{i}{2}\sum_{j=1}^{2}\int\frac{1}{\lambda_j}&[\bar\varphi_j(x)\,\gamma^\mu\,\partial_\mu\varphi_j(x) - \partial_\mu\bar\varphi_j(x)\,\gamma^\mu\,\varphi_j(x)]\,\mathrm{d}^3x\\ -\frac{1}{4}g\int&\left\{\left[\sum_{j=1}^{2}\bar\varphi_j(x)\right]\left[\sum_{k=1}^{2}\varphi_h(x)\right]\right\}^2\mathrm{d}^3x\,. \qquad (1.11)\end{aligned}$$

Hence we have the fact that Hamiltonians which are equivalent under subsidiary conditions possess Lagrangians which differ from each other.

2. Symmetry Breaking by Quantization

By dropping a four divergence term in the Lagrangian (1.2) we can rewrite it in the form

$$L[\psi] = i \int \bar{\psi}(x)\, \gamma^\mu\, \partial_\mu \psi(x)\, \mathrm{d}^3x - g \int [\bar{\psi}(x)\, \psi(x)]^2\, \mathrm{d}^3x\,. \tag{2.1}$$

The canonical conjugate field momenta of the original ψ-field follow then from (2.1) by

$$\pi(x) = \frac{\delta L}{\delta(\partial\psi(x)/\partial t)} = i\, \psi(x)^+ \tag{2.2}$$

and the anticommutator reads

$$[\psi(\boldsymbol{r}', t)^+, \psi(\boldsymbol{r}, t)]_+ = \mathbf{1}\, \delta(\boldsymbol{r}-\boldsymbol{r}')\,. \tag{2.3}$$

From (2.3) the anticommutator for the parity transform (1.6) can be derived. This yields

$$[\xi(\boldsymbol{r}', t)^+, \xi(\boldsymbol{r}, t)]_+ = \mathbf{1}\, \delta(\boldsymbol{r}-\boldsymbol{r}')\,. \tag{2.4}$$

On the other hand, if we start from the Lagrangian (1.10) or (1.11) resp., we obtain for the anticommutators

$$[\varphi_j(\boldsymbol{r}', t)^+, \varphi_j(\boldsymbol{r}, t)]_+ = \lambda_j\, \mathbf{1}\, \delta(\boldsymbol{r}-\boldsymbol{r}')\,, \tag{2.5}$$

i.e. these anticommutators are not compatible with (2.3) and (2.4). This means that quantization destroys the subsidiary condition (1.6). While on the classical level both Hamiltonians are equivalent, this is not the case on the quantum level due to the impossibility of fulfilling (1.6). This has further consequences. The original Hamiltonian and Lagrangian allows for a global gauge group

$$\psi'(x) = e^{i\alpha q}\, \psi(x) \tag{2.6}$$

which leave both expressions forminvariant. From this it follows that the particle number operator N must commute with $H[\psi]$, i.e. leads to the definition of the particle number as a good quantum number, where $N[\psi]$ is given by

$$N[\psi] := \int \bar{\psi}(\boldsymbol{r})\, \psi(\boldsymbol{r})\, \mathrm{d}^3r \tag{2.7}$$

with $[N, H]_- = 0$. If the subsidiary condition (1.6) is satisfied we obtain

$$N[\psi, \xi] = \tfrac{1}{2} \int \bar{\psi}(\boldsymbol{r})\, \psi(\boldsymbol{r})\, \mathrm{d}^3r + \tfrac{1}{2} \int \bar{\xi}(\boldsymbol{r})\, \xi(\boldsymbol{r})\, \mathrm{d}^3r \tag{2.8}$$

and therefore

$$[N[\psi], H[\psi]]_- = [N[\psi, \xi], H[\psi, \xi]]_- = 0\,. \tag{2.9}$$

If by quantization the subsidiary condition (1.6) cannot be satisfied, then for $\xi \neq i\, \gamma_4\, \psi$

$$\begin{aligned} &[N[\psi], H[\psi, \xi]]_- \neq 0\,, \\ &[N[\xi], H[\psi, \xi]]_- \neq 0\,, \end{aligned} \tag{2.10}$$

i.e. we have no longer conserved particle numbers with respect to the ψ- and ξ-particles. In this case it is convenient to use the grand canonical Hamiltonian

$$K[\psi, \xi] := H[\psi, \xi] - \mu_\psi\, N[\psi] + \mu_\xi\, N[\xi]\,. \tag{2.11}$$

The corresponding grand canonical Lagrangian is then given by

$$\begin{aligned} \mathbb{L}[\psi, \xi] := \frac{i}{2} \sum_{j=1}^{2} \int \frac{1}{\lambda_j} [&\bar{\varphi}_j(x)\, \gamma^\mu\, \partial_\mu \varphi_j(x) \\ &- \partial_\mu \bar{\varphi}_j(x)\, \gamma^\mu\, \varphi_j(x) - \mu_j\, \bar{\varphi}_j(x)\, \varphi_j(x)]\, \mathrm{d}^3x \\ &- \frac{1}{4} g \int \left\{ \left[\sum_{j=1}^{2} \bar{\varphi}_j(x) \right] \left[\sum_{k=1}^{2} \varphi_h(x) \right] \right\}^2 \mathrm{d}^3x\,. \end{aligned} \tag{2.12}$$

Defining

$$\Psi(x) := \varphi_1(x) + \varphi_2(x) \tag{2.13}$$

it was shown in previous papers [29, 32] that the Lagrangian (2.12) leads to the renormalizable equation

$$(i\, \gamma^\mu\, \partial_\mu - \mu_1)\, (i\, \gamma^\mu\, \partial_\mu - \mu_2)\, \Psi(x) = \frac{1}{4} g\, \Psi(x)\, \bar{\Psi}(x)\, \Psi(x)\,. \tag{2.14}$$

From this it follows that the nonrenormalizable Eq. (1.1) and the renormalizable Eq. (2.14) are inequivalent quantum representations which are derived from equivalent classical Hamiltonians.

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